Vectors in Two Dimensions

PH11-8

ORIENTATION

Lesson goal: build accurate physics fluency for vectors in two dimensions and use that fluency to support clear HSC-style scientific writing.

This page is materialised into the MentorMind course shell from existing teaching, textbook, and eduKG material. Use it as the main lesson surface; use the tutor for targeted repair, worked examples, and concise writing feedback.

SYLLABUS INQUIRY QUESTION

- How is the motion of an object moving in a straight line described and predicted?

From The Feynman Lectures on Physics, Vol I, Chapter 11:

Vector addition is not a trick; it is a statement about how nature combines directions. Treating velocities like scalars loses the geometry of motion.

LEARNING OBJECTIVES

- Represent vectors using components and diagrams.
- Resolve vectors into perpendicular components.
- Add and subtract vectors in two dimensions.
- Apply vector methods to velocity and displacement.

CONTENT

Vector components

A vector $\vec{R}$ at angle θ from the horizontal has components:

$$R_x = R\cos\theta, \quad R_y = R\sin\theta$$

The angle is always measured from the **positive x-axis** (east direction) unless otherwise specified.

Interactive: Vector Resolution

The diagram below shows a vector being resolved into its x and y components:

Vector addition

Graphical methods (head-to-tail) and component methods both yield the resultant. The **component method** is preferred for calculations.

Interactive: Vector Addition (Tail-to-Head)

Add two vectors using the tail-to-head method. The resultant (dashed) connects the start to the end.

Method using components:

1. Resolve each vector into x and y components
2. Add all x components: $R_x = A_x + B_x$
3. Add all y components: $R_y = A_y + B_y$
4. Find magnitude: $R = \sqrt{R_x^2 + R_y^2}$
5. Find direction: $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

Resultant magnitude and direction

$$R = \sqrt{R_x^2 + R_y^2}, \quad \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Watch the quadrant! If $R_x < 0$, the angle from $\tan^{-1}$ needs adjustment (add 180 degrees).

Interactive: Three-Vector Addition

When adding more than two vectors, the component method becomes essential:

WORKED EXAMPLES

Example 1: Resolve a vector

A displacement of 50 m is directed 30 degrees north of east.

Solution:

1. $R_x = 50\cos(30 degrees) = 50 \times 0.866 = 43.3$ m (east)
2. $R_y = 50\sin(30 degrees) = 50 \times 0.5 = 25.0$ m (north)
3. Components describe the east and north parts of the motion.

Example 2: Add two vectors

A walker goes 3.0 m east, then 4.0 m north.

Solution:

1. Components: $R_x = 3.0$ m, $R_y = 4.0$ m
2. Resultant magnitude: $R = \sqrt{3.0^2 + 4.0^2} = \sqrt{25} = 5.0$ m
3. Direction: $\theta = \tan^{-1}(4.0/3.0) = 53 degrees$ north of east

Example 3: Subtract vectors (relative motion)

A boat's velocity is 6.0 m/s east. The current is 2.0 m/s north. Find the boat's velocity relative to the water.

Solution:

1. $\vec{v}_{bw} = \vec{v}_{be} - \vec{v}_{we}$
2. Components: $v_x = 6.0$ m/s, $v_y = -2.0$ m/s (subtract current)
3. Speed: $\sqrt{6.0^2 + 2.0^2} = 6.3$ m/s
4. Direction: $\tan^{-1}(2.0/6.0) = 18 degrees$ south of east

Example 4: Find components from magnitude and direction

A force of 25 N acts at 60 degrees above the horizontal.

Solution:

1. $F_x = 25\cos(60 degrees) = 12.5$ N (horizontal)
2. $F_y = 25\sin(60 degrees) = 21.7$ N (vertical)

COMMON MISCONCEPTIONS

- **Misconception:** Components are always equal to the magnitude. **Correction:** Components depend on direction; only at 45 degrees are they equal.

- **Misconception:** Vector addition is the same as adding magnitudes. **Correction:** Directions change the result; 3 m east + 4 m north ≠ 7 m.
- **Misconception:** A negative component means the vector is negative. **Correction:** It only indicates direction along the axis.
- **Misconception:** $\tan^{-1}$ always gives the correct angle. **Correction:** You must check the quadrant based on component signs.

PRACTICE QUESTIONS

Easy (2 marks)

Resolve a 10 m displacement at 60 degrees above the horizontal into components.

- Correct cosine and sine components (2)

Answer:

- $x = 10\cos(60 degrees) = 5.0\text{ m}$
- $y = 10\sin(60 degrees) = 8.7\text{ m}$

Medium (4 marks)

Two forces act on a point: 8 N east and 6 N north. Find the resultant magnitude and direction.

- Correct components and resultant magnitude (2)
- Correct direction (2)

Answer:

- $R = \sqrt{8^2 + 6^2} = \sqrt{100} = 10\text{ N}$
- $\theta = \tan^{-1}(6/8) = 37 degrees$ north of east

Hard (5 marks)

An aircraft travels 200 km/h north relative to the air. A wind of 60 km/h blows east. Find the ground velocity and its direction.

- Correct vector model (1)
- Components and magnitude (2)
- Correct direction statement (2)

Solution:

The ground velocity is the sum of air velocity and wind velocity:

- $v_x = 0 + 60 = 60$ km/h (east)
- $v_y = 200 + 0 = 200$ km/h (north)

Ground speed:

- $v = \sqrt{60^2 + 200^2} = \sqrt{43600} = 209$ km/h

Direction:

- $\theta = \tan^{-1}(60/200) = 17 degrees$ east of north

Answer: 209 km/h at 17 degrees east of north

MULTIPLE CHOICE QUESTIONS

Test your understanding with these interactive questions:

SUMMARY

- Components encode vector direction in x and y axes.
- Vector addition can be solved with components and Pythagoras.
- Direction is found using inverse tangent with correct quadrant.
- Vector methods underpin two-dimensional motion.

SELF-ASSESSMENT

Check your understanding:

After studying this section, you should be able to:

- ☐ Resolve a vector into perpendicular components
- ☐ Add two or more vectors using components
- ☐ Calculate resultant magnitude using Pythagoras
- ☐ Determine direction using inverse tangent
- ☐ Check which quadrant the resultant lies in

SCIENTIFIC WRITING AND EXAM SUPPORT

When answering questions from this lesson, separate:

- the physical quantity being discussed,
- the model or law being applied,
- the mathematical relationship, including units,
- the conclusion in words.

For explanation questions, write in the pattern: **claim -> physics reason -> consequence**. For calculation questions, state the formula, substitute with units, calculate, then interpret the answer.

MAINTENANCE LOOP

One-minute retrieval:

1. State the key law, model, or relationship used in this lesson.
2. Identify one common misconception that would lead to a wrong answer.
3. Write one sentence that links the calculation or evidence back to the physical meaning.

STUDENT WORKING